

**Modified Enlarged 24pt**  
**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Monday 18 October 2021 – Afternoon**

**A Level Mathematics B (MEI)**

**H640/03 Pure Mathematics and  
Comprehension**

**Time allowed: 2 hours  
plus your additional time allowance**

**YOU MUST HAVE:**  
**the Printed Answer Booklet**  
**the Insert**  
**a scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**



# **INSTRUCTIONS**

**Use black ink. You can use an HB pencil, but only for graphs and diagrams.**

**Write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.**

**Fill in the boxes on the front of the Printed Answer Booklet.**

**Answer ALL the questions.**

**Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.**

**Give your final answers to a degree of accuracy that is appropriate to the context.**

**Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.**

## **INFORMATION**

**The total mark for this paper is 75.**

**The marks for each question are shown in brackets [ ].**

## **ADVICE**

**Read each question carefully before you start your answer.**

# Formulae A Level Mathematics B (MEI) (H640)

## Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

## Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

## Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

## Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule  $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

## Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

## Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts  $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

## Small angle approximations

$\sin \theta \approx \theta$ ,  $\cos \theta \approx 1 - \frac{1}{2}\theta^2$ ,  $\tan \theta \approx \theta$  where  $\theta$  is measured in radians

## Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$(A \pm B \neq (k + \frac{1}{2})\pi)$$

## Numerical methods

Trapezium rule:

$$\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\},$$

$$\text{where } h = \frac{b-a}{n}$$

The Newton-Raphson iteration for solving

$$f(x) = 0: x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

## Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B)$$

$$\text{OR } P(A | B) = \frac{P(A \cap B)}{P(B)}$$

## Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where}$$

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation,  $s = \sqrt{\text{variance}}$

## The binomial distribution

If  $X \sim B(n, p)$  then  $P(X = r) = {}^nC_r p^r q^{n-r}$   
where  $q = 1 - p$

Mean of  $X$  is  $np$

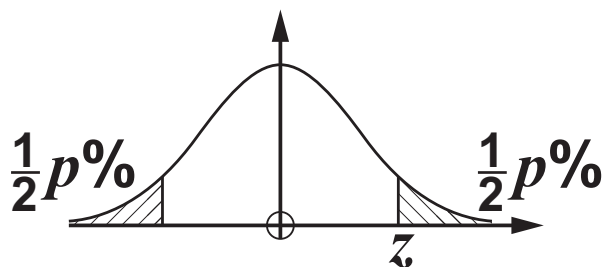
## Hypothesis testing for the mean of a Normal distribution

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$$

## Percentage points of the Normal distribution

$p$	10	5	2	1
$z$	1.645	1.960	2.326	2.576





# Kinematics

## Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

## Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

**Answer ALL the questions.**

**SECTION A (60 marks)**

- 1 (a) Express  $x^2 + 8x + 2$  in the form  $(x + a)^2 + b$ . [2]**
- (b) Write down the coordinates of the turning point of the curve  $y = x^2 + 8x + 2$ . [1]**
- (c) State the transformation(s) which map(s) the curve  $y = x^2$  onto the curve  $y = x^2 + 8x + 2$ . [2]**
- 2 Solve the equation  $\sin 2x = 0.3$  for  $0^\circ \leq x \leq 180^\circ$ . Give your answer(s) correct to 1 decimal place. [2]**

- 3 (a) Determine, in terms of  $k$ , the coordinates of the point where the lines with the following equations intersect.**

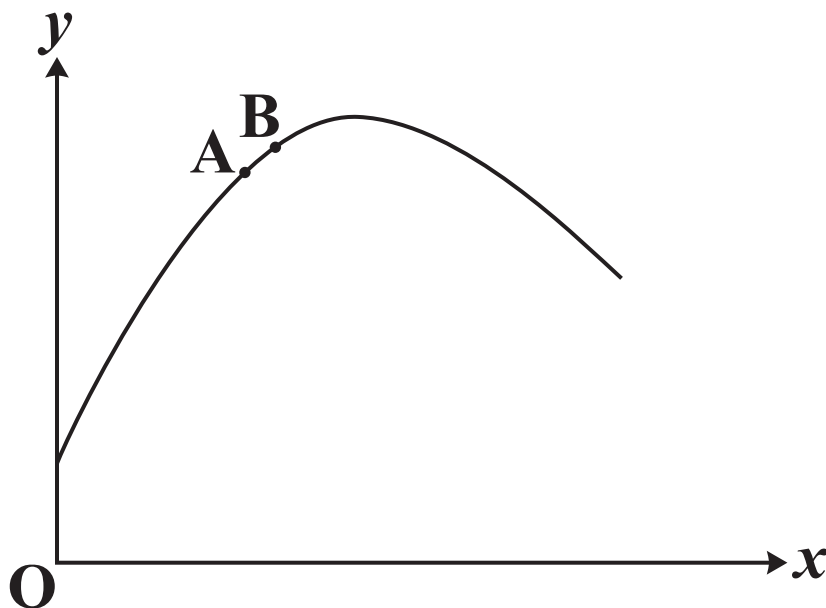
$$x + y = k$$

$$2x - y = 1 \quad [3]$$

- (b) Determine, in terms of  $k$ , the coordinates of the points where the line  $x + y = k$  crosses the curve  $y = x^2 + k$ . [4]**

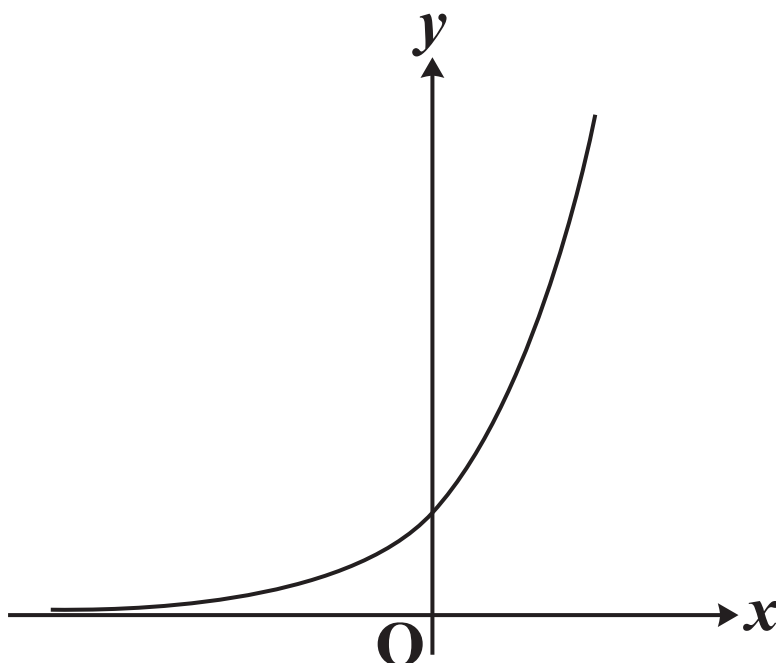
- 4 The diagram shows points A and B on the curve  $y = \left(\frac{x}{4}\right)^{-x}$ .**

**The  $x$ -coordinate of A is 1 and the  $x$ -coordinate of B is 1.1.**



- (a) Find the gradient of chord AB. Give your answer correct to 2 decimal places. [2]**
- (b) Give the  $x$ -coordinate of a point C on the curve such that the gradient of chord AC is a better approximation to the gradient of the tangent to the curve at A. [1]**

- 5 (a) The diagram shows the curve  $y = e^x$ .**



**On the axes in the Printed Answer Booklet, sketch graphs of**

- (i)  $\frac{dy}{dx}$  against  $x$ , [1]**
- (ii)  $\frac{dy}{dx}$  against  $y$ . [2]**

- (b) Wolves were introduced to Yellowstone National Park in 1995.**

**The population of wolves,  $y$ , is modelled by the equation**

$$y = Ae^{kt},$$

**where  $A$  and  $k$  are constants and  $t$  is the number of years after 1995.**

- (i) Give a reason why this model might be suitable for the population of wolves. [1]**

- (ii) When  $t = 0$ ,  $y = 21$  and when  $t = 1$ ,  $y = 51$ .**

**Find values of  $A$  and  $k$  consistent with the data. [3]**

- (iii) Give a reason why the model will not be a good predictor of wolf populations many years after 1995. [1]**

- 6 In this question you must show detailed reasoning.**

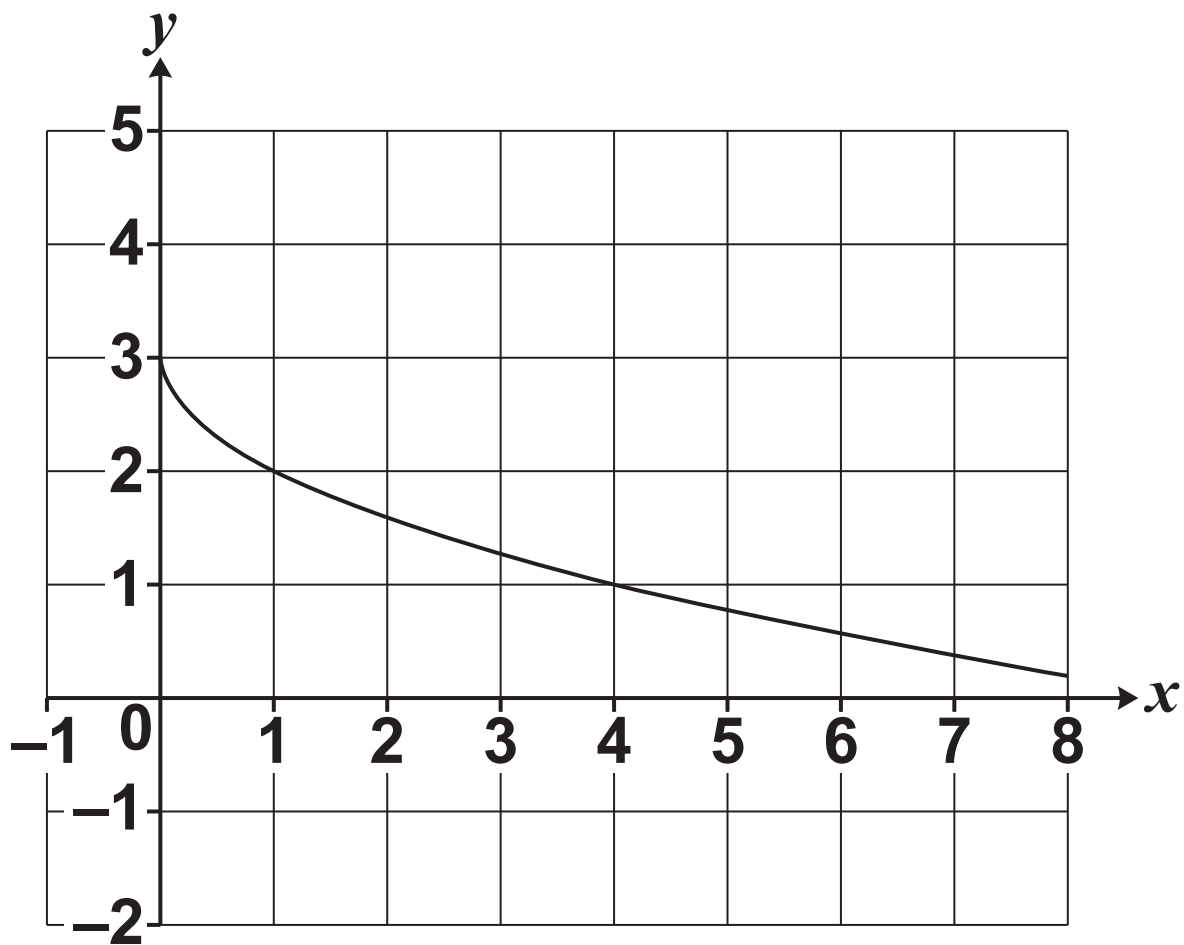
**Show that  $\sum_{r=1}^3 \frac{1}{\sqrt{r+1} + \sqrt{r}} = 1$ . [4]**

**7 Determine  $\int x \cos 2x \, dx$ . [3]**

- 8 For a particular value of  $a$ , the curve  $y = \frac{a}{x^2}$  passes through the point (3, 1).**

**Find the coordinates of all the other points on the curve where both the  $x$ -coordinate and the  $y$ -coordinate are integers. [3]**

- 9 The diagram shows the curve**  
 **$y = 3 - \sqrt{x}$ .**



- (a) Draw the line  $y = 5x - 1$  on the copy of the diagram in the Printed Answer Booklet. [1]**



**(b) In this question you must show detailed reasoning.**

**Determine the exact area of the region bounded by the curve  $y = 3 - \sqrt{x}$ , the lines  $y = 5x - 1$  and  $x = 4$  and the  $x$ -axis. [10]**

**10 (a) Express  $\frac{1}{(4x+1)(x+1)}$  in partial fractions. [3]**

**(b) A curve passes through the point (0, 2) and satisfies the differential equation**

$$\frac{dy}{dx} = \frac{y}{(4x+1)(x+1)},$$

**for  $x > -\frac{1}{4}$ .**

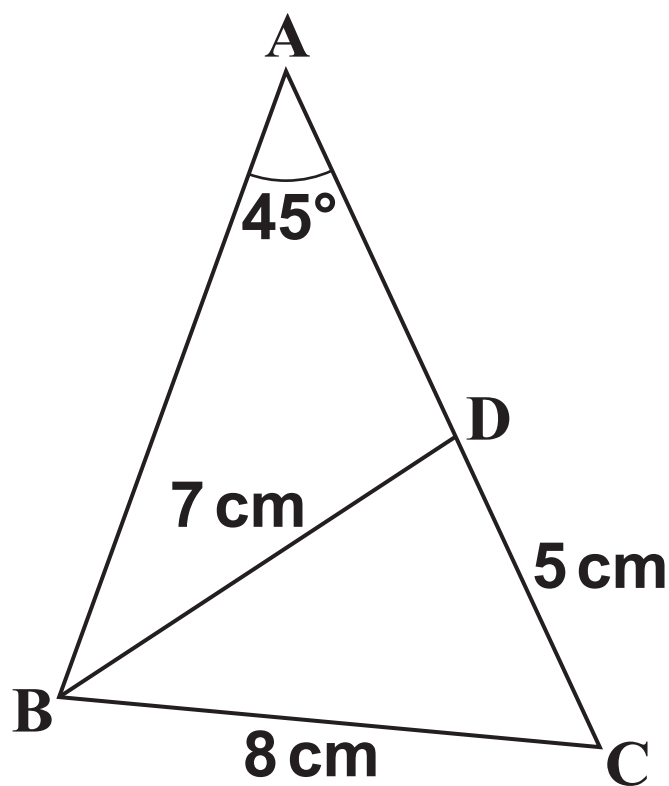
**Show by integration that**

**$y = A\left(\frac{4x+1}{x+1}\right)^B$  where  $A$  and  $B$  are constants to be determined. [6]**

**11 In this question you must show detailed reasoning.**

**The diagram shows triangle ABC, with  $BC = 8\text{ cm}$  and angle  $BAC = 45^\circ$ .**

**The point D on AC is such that  $DC = 5\text{ cm}$  and  $BD = 7\text{ cm}$ .**



**Determine the exact length of AB. [5]**

**Answer ALL the questions.**

**SECTION B (15 marks)**

**The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.**

**12 Show that  $\beta = \arctan\left(\frac{1}{3}\right)$ , as given in line 24. [3]**

**13 (a) Use triangle ABE in FIG. C2 to show that  $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$ , as given in line 51. [1]**

**(b) Sketch the graph of  $y = \arctan x$ . [1]**

**(c) What property of the arctan function ensures that  $y > \frac{1}{x} \Rightarrow \arctan y > \arctan\left(\frac{1}{x}\right)$ , as given in line 52? [1]**

**14 (a) Show that**

$$\begin{aligned} \arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) &= \\ \arctan\left(\frac{1}{n}\right) &\Rightarrow \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \\ \arctan 1. & \text{ [1]} \end{aligned}$$

**(b) Use the arctan addition formula in lines 37–38 to show that**

$$\begin{aligned} \arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) &= \\ \arctan\left(\frac{1}{n}\right), & \text{ as given in} \\ \text{lines 73–74.} & \text{ [4]} \end{aligned}$$

**15 Prove that**

$$\arctan 1 + \arctan 2 + \arctan 3 = \pi, \text{ as given in line 77. [4]}$$

**END OF QUESTION PAPER**

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